The physics of organ blowing - supplement

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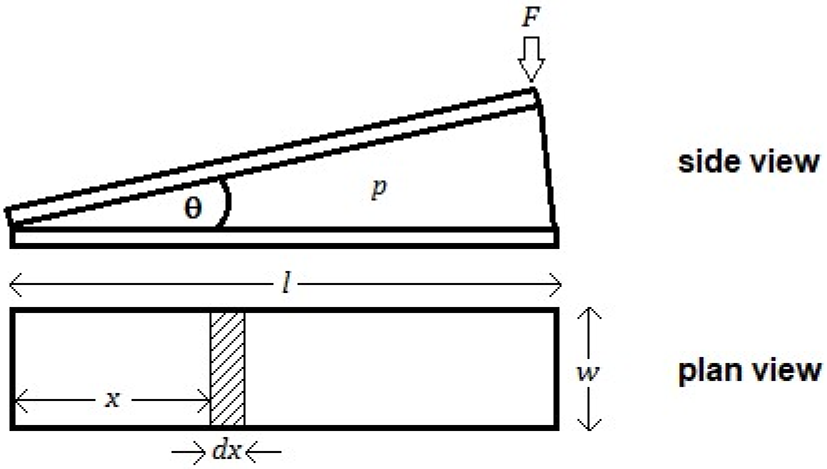
This is a supplementary note to the article on my website entitled ‘The physics of organ blowing’[1]. It contains some mathematical details which were deemed too involved for inclusion in the article itself. They are relevant to two types of hinged bellows: those with a rectangular top board and those with a triangular one. The simpler case of a horizontal bellows with no hinge, more frequently used as a wind reservoir rather than a feeder, was considered in the main article.

Hinged bellows – rectangular top



The type of bellows in question is sketched above and it was often used as a feeder bellows supplying the pipes directly, or indirectly by pressurising a reservoir which then fed the pipework. The force is assumed to be exerted by the person doing the blowing at the edge of the top board remote from the hinge and it is denoted by *F*. Note that the pressure will change if the force acts at a different point along the board. If the blower stands bodily on the board, by Newton’s second law of motion the value of the force will be *Mg* where *M* is the blower’s mass and *g* the acceleration due to gravity. Even if not physically standing on the board, blowers cannot exert a downwards force greater than this except by using a lever, otherwise they would lift themselves off the ground.

It is more intuitive to calculate *F* by taking the internal pressure *p* as a given, rather than to approach the problem the other way round i.e. by calculating the pressure due to a given applied force. In other words, we shall calculate the total counterbalancing force measured at the right hand edge of the top board assuming air is blown into the bellows to give a pressure *p*. However both approaches give the same result of course.

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[1] *www.pykett.org.uk/physics-of-organ-blowing.htm*

The force exerted by the internal pressure *p* on the shaded infinitesimally narrow strip of width *dx* is the product of pressure and area i.e. *pwdx*. Measured at the right hand edge of the top board remote from the hinge, the contribution of this force increment to the total force varies owing to the leverage effect due to the hinge. The situation is in fact a type 3 lever where the incremental force measured at the right hand edge of the board increases linearly as the strip moves away from the hinge. The force increment at the board edge due to a strip at distance *x* from the hinge, as drawn in the lower diagram, is thus *pwdx.x/l* where *x/l* is the mechanical advantage or leverage ratio. Therefore, to get the total force *F* at this point we integrate the force increment with respect to *x* over the length *l* of the board:

$$F = \int_0^l pwdx.\frac{x}{l}$$

$$= \frac{pw}{l}\int_0^l x\,dx$$

$$= \frac{pw}{l}.\frac{l^2}{2}$$

$$= \frac{pwl}{2}$$

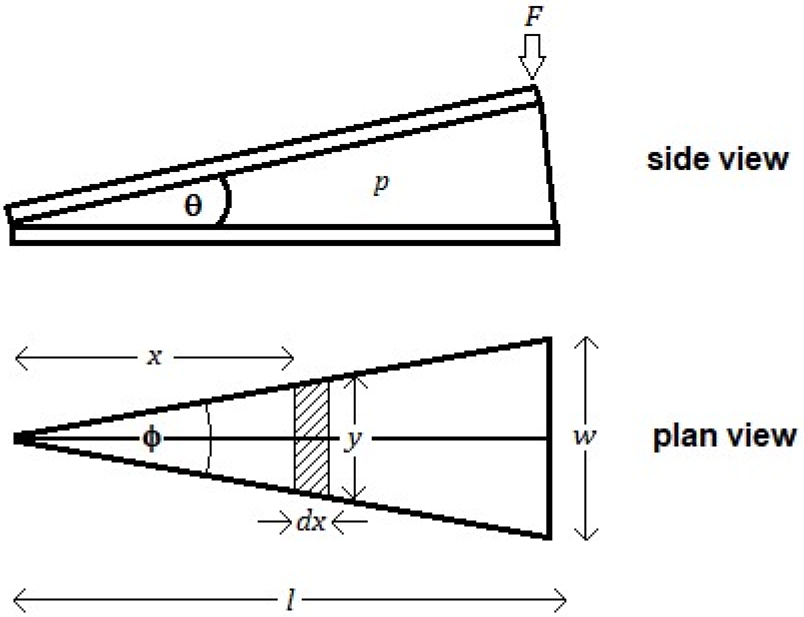
$$\therefore p = \frac{2F}{wl}$$

This is the pressure inside the bellows produced by a force *F* at its right hand edge as shown in the diagram. Note that it is twice that produced by the same force applied to the top board of a horizontal bellows without a hinge. This is because the applied force is effectively amplified by the leverage effect towards the hinge and it therefore results in an increased pressure overall.

Because the force *F* is shown acting vertically downwards the wind pressure will increase as the bellows collapses, following the value of $\cos\theta$. However in this analysis the effect has been ignored since for small angles, say less than 20 degrees, $\cos\theta \approx 1$.

By inspection, the internal volume of the bellows is half that of the horizontal bellows. Therefore the volume of air expelled has reduced by the same factor of two that the pressure has increased relative to a horizontal bellows. This means that more bellows and blowers would be required to get the same air flow if the other dimensions remain the same. (These figures for the pressure increase and volume reduction are approximate because practical bellows would use folded leather sides which affect both internal pressure and volume).

Hinged bellows – triangular top



This type of bellows is sketched above and, like that which used a rectangular top board discussed above, it was sometimes used as a feeder bellows either supplying the pipes directly or by first charging a reservoir. This or other shapes having a tapered top board were also commonly used for domestic purposes. In this example the top board is now an isosceles triangle defining the angle ϕ rather than a rectangle. Also as before, the force is assumed to be exerted by the blower at the edge of the top board remote from the hinge and it is denoted by F. Note that the pressure will vary if the force acts at a different point along the board. Repeating what was said above, if the blower stands bodily on the board, by Newton's second law of motion the value of the force will be Mg where M is the blower's mass and g the acceleration due to gravity. Even if not physically standing on the board, blowers cannot exert a downwards force greater than this except by using a lever, otherwise they would lift themselves off the ground.

Following the previous approach, it is more intuitive to calculate F by taking the internal pressure p as a given, rather than to address the problem the other way round i.e. by calculating the pressure due to a given applied force. In other words, we shall calculate the total counterbalancing force measured at the right hand edge of the top board assuming air is blown into the bellows to give a pressure p. However both approaches give the same result.

The force exerted by the internal pressure p on the shaded infinitesimally narrow strip of width dx and height y in the lower diagram is the product of pressure and area i.e. $pydx$. Measured at the right hand edge of the top board remote from the hinge, the contribution of this force increment to the total force varies owing to the leverage effect due to the hinge. The situation is in fact a type 3 lever where the incremental force measured at the right hand edge of the board increases linearly as the strip moves away from the hinge. The force increment at the board edge due to a strip at distance *x* from the hinge, as drawn in the lower diagram, is thus *pydx.x/l* where *x/l* is the mechanical advantage or leverage ratio. But, unlike the previous case where the height of the strip was constant and equal to w, y is now an additional variable given in terms of *x* by:

$$y = 2x.\tan\frac{\emptyset}{2}$$

Therefore to get the total force *F* we again integrate the force increment with respect to *x* over the length *l* of the board:

$$F = \int_0^l (p.dx).(2x\tan\tfrac{\emptyset}{2}).\frac{x}{l}$$

$$= \frac{2p}{l}\tan\frac{\emptyset}{2}\int_0^l x^2\,dx$$

$$= \frac{2pl^2}{3}\tan\frac{\emptyset}{2}$$

$$\therefore p = \frac{3F}{2\,l^2\tan\frac{\emptyset}{2}}$$

$$\text{But}\tan\frac{\emptyset}{2} = \frac{w}{2l}$$

$$\therefore p = \frac{3F}{wl}$$

Note the interesting fact that this result is independent of the angle ϕ and thus of the shape of the triangular top board (whether it is broad or narrow). Hence the pressure is now three times that which would be produced by the same force applied to the top board of a horizontal bellows without a hinge, and 50% more than for the previous case of a bellows with a hinged rectangular board. The latter effect is because the applied force results in an additional pressure increase since the area of the top board has been reduced.

Because the force *F* is shown acting vertically downwards the wind pressure will increase as the bellows collapses, following the value of cosθ. However in this analysis the effect has been ignored since for small angles, say less than 20 degrees, cosθ ≈ 1.

Using a similar approach to that used in calculating the pressure, it can be shown that the internal volume of the bellows, and thus the volume of air expelled, has reduced by the same factor of three that the pressure has increased relative to a horizontal bellows. This means that more bellows and blowers would be required to get the same air flow if the other dimensions remain the same. (These figures for the pressure increase and volume reduction are approximate because practical bellows would use folded leather sides which affect both internal pressure and volume).